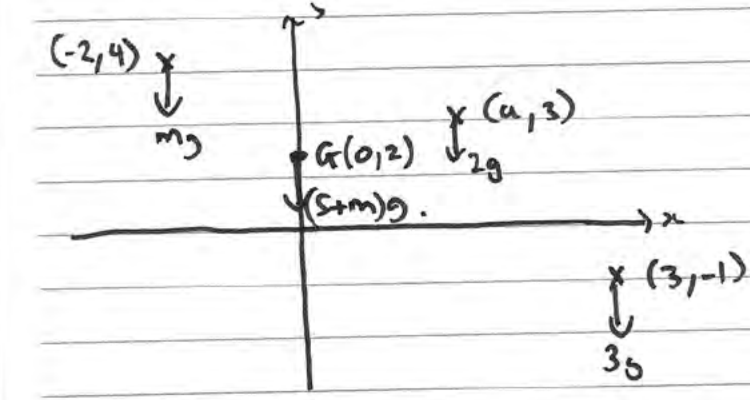
M2 June 2013 un original.

1. Three particles of masses 2 kg, 3 kg and m kg are positioned at the points w
$(a, 3)$, $(3, -1)$ and $(-2, 4)$ respectively. Given that the centre of mass of the pa
the point with coordinates $(0, 2)$, find

(a) the value of m, **(4)**

(b) the value of a. **(4)**



a) x: $3g \times -1 + 2g \times 3 + mg \times 4 = (5+m)g \times 2$

$\Rightarrow 3\cancel{g} + 4m\cancel{g} = 10\cancel{g} + 2m\cancel{g}$

$2m = 7 \quad \therefore \underline{m = 3.5\text{kg}}$

b) y

$3.5\cancel{g} \times -2 + 2\cancel{g} \times a + 3\cancel{g} \times 3 = 0$

$-7 + 2a + 9 = 0 \quad \Rightarrow 2a = -2 \quad \therefore \underline{a = -1}$

2. A car has mass 1200 kg. The maximum power of the car's engine is 32 k… resis… to motion due to non-gravitational forces is modelled as a force of cons… 800 N. When the car is travelling on a horizontal road at constant speed … engine of the car is working at maximum power.

(a) Find the value of V.

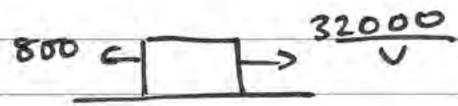
(3)

The car now travels downhill on a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \dfrac{1}{40}$. The resistance to motion due to non-gravitational forces is still modelled as a force of constant magnitude 800 N. Given that the engine of the car is again working at maximum power,

(b) find the acceleration of the car when its speed is 20 m s^{-1}.

(4)

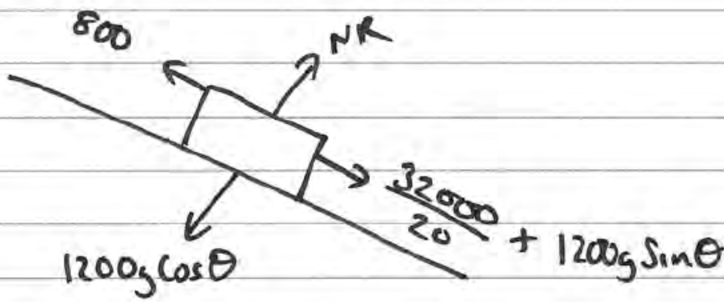
a)



$R_f = 0 \Rightarrow \dfrac{3200\cancel{0}}{V} = 80\cancel{0}$

$\therefore V = 40\ ms^{-1}$

b)



$R_f \searrow = ma \Rightarrow 30g + 1600 - 800 = 1200a$

$\therefore a = 0.91\ ms^{-2}$ (2sf)

3 A particle P of mass 0.25 kg moves under the action of a single force ... ns. ... t seconds, the velocity of P is $\mathbf{v}$ m s^{-1}, where

$$\mathbf{v} = (2 - 4t)\mathbf{i} + (t^2 + 2t)\mathbf{j}$$

When $t = 0$, P is at the point with position vector $(2\mathbf{i} - 4\mathbf{j})$ m with respect to a fixed ... O. When $t = 3$, P is at the point A. Find

(a) the momentum of P when $t = 3$,
(2)

(b) the magnitude of $\mathbf{F}$ when $t = 3$,
(6)

(c) the position vector of A.
(5)

a) $v = \begin{pmatrix} 2-4t \\ t^2+2t \end{pmatrix}$ $\quad t=3 \Rightarrow v = \begin{pmatrix} -10 \\ 15 \end{pmatrix}$

momentum $= mv = \frac{1}{4}\begin{pmatrix} -10 \\ 15 \end{pmatrix} = \begin{pmatrix} -2.5 \\ 3.75 \end{pmatrix}$

b) $F = ma$ $\quad a = \frac{dv}{dt} = \begin{pmatrix} -4 \\ 2t+2 \end{pmatrix}$ $\quad t=3 \quad a = \begin{pmatrix} -4 \\ 8 \end{pmatrix}$

$\therefore F = \frac{1}{4}\begin{pmatrix} -4 \\ 8 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\quad |F| = \sqrt{1^2+2^2} = \underline{\sqrt{5}\,N}$

c) $s = \int v\,dt = \begin{pmatrix} 2t-2t^2+c_1 \\ \frac{1}{3}t^3+t^2+c_2 \end{pmatrix}$ $\quad$ at $t=0 \quad s = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

$\therefore c_1 = 2, \; c_2 = -4$

$s = \begin{pmatrix} 2t-2t^2+2 \\ \frac{1}{3}t^3+t^2-4 \end{pmatrix}$ $\quad t=3 \quad \therefore A = \begin{pmatrix} -10 \\ 14 \end{pmatrix}$

4.

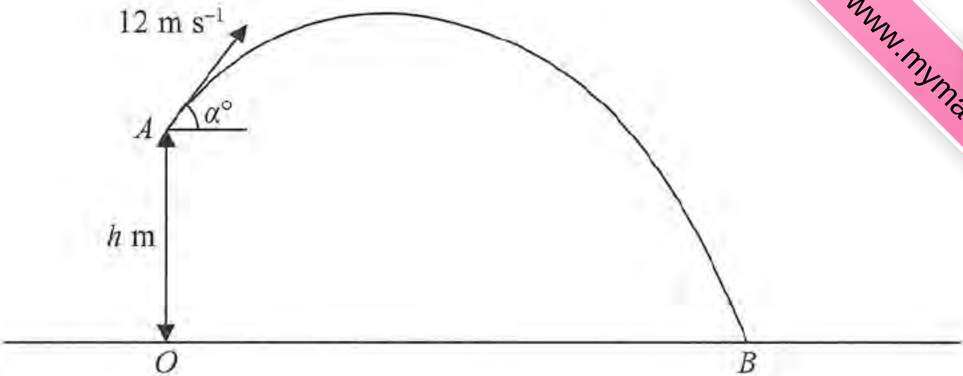


Figure 1

The points O and B are on horizontal ground. The point A is h metres vertically above O. A particle P is projected from A with speed 12 m s^{-1} at an angle $\alpha°$ to the horizontal. The particle moves freely under gravity and hits the ground at B, as shown in Figure 1. The speed of P immediately before it hits the ground is 15 m s^{-1}.

(a) By considering energy, find the value of h.

(4)

Given that 1.5 s after it is projected from A, P is at a point 4 m above the level of A, find

(b) the value of α,

(3)

(c) the direction of motion of P immediately before it reaches B.

(3)

a) Gain in KE = loss in PE

$\frac{1}{2}\cancel{m}(15^2 - 12^2) = \cancel{m}gh \quad \therefore h = \frac{81}{2g} \simeq 4.1\text{m}$

b) $(v\uparrow)$

$s = 4$

$u = 12\sin\alpha$

v

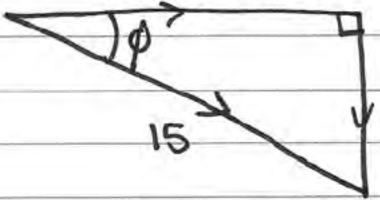
$a = -9.8$

$t = 1.5$

$s = ut + \frac{1}{2}at^2 \Rightarrow 4 = 18\sin\alpha - 4$

$\therefore \sin\alpha = \dfrac{15.025}{18} \Rightarrow \alpha = 56.6$

c) $12\cos\alpha = 6.608$



$\phi = \cos^{-1} \dfrac{6.608\ldots}{15}$

$\phi = 63.9°$ below horizontal

5.

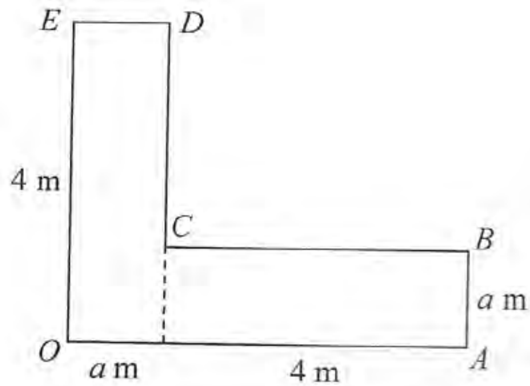


Figure 2

The uniform L-shaped lamina *OABCDE*, shown in Figure 2, is made from two identical rectangles. Each rectangle is 4 metres long and a metres wide. Giving each answer in terms of a, find the distance of the centre of mass of the lamina from

(a) OE,

(4)

(b) OA.

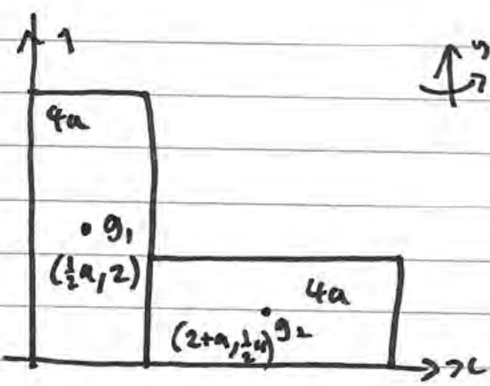
(4)

The lamina is freely suspended from O and hangs in equilibrium with OE at an angle θ to the downward vertical through O, where $\tan\theta = \dfrac{4}{3}$.

(c) Find the value of a.

(4)

a)



$$4a \times \tfrac{1}{2}a + 4a \times (2+a) = 8a \times \bar{x}$$

$$2a^{\cancel{2}} + 8\cancel{a} + 4a^{\cancel{2}} = 8\cancel{a}\,\bar{x}$$

$$6a + 8 = 8\bar{x}$$

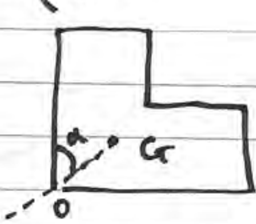
$$\bar{x} = \tfrac{3}{4}a + 1$$

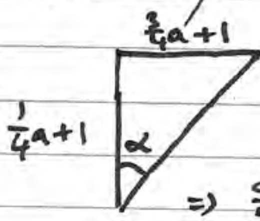
b)

$$4a \times 2 + 4a \times \tfrac{1}{2}a = 8a \times \bar{y}$$

$$8\cancel{a} + 2a^{\cancel{2}} = 8\cancel{a}\,\bar{y} \qquad \bar{y} = \tfrac{1}{4}a + 1$$

c)





$$\tan\alpha = \frac{\tfrac{3}{4}a+1}{\tfrac{1}{4}a+1} = \frac{4}{3}$$

$$\Rightarrow \tfrac{9}{4}a + 3 = a + 4 \quad \Rightarrow \tfrac{5}{4}a = 1 \quad \therefore a = \tfrac{4}{5}$$

6.

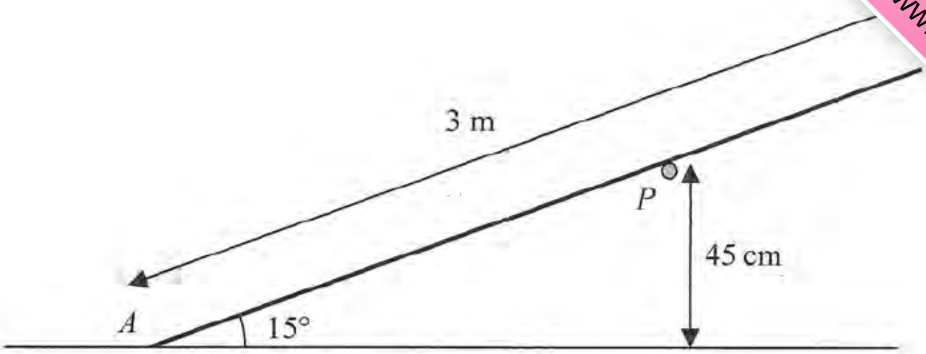


Figure 3

A uniform rod AB has weight 30 N and length 3 m. The rod rests in equilibrium on a rough horizontal peg P with its end A on smooth horizontal ground. The rod is in a vertical plane perpendicular to the peg. The rod is inclined at 15° to the ground and the point of contact between the peg and the rod is 45 cm above the ground, as shown in Figure 3.

(a) Show that the normal reaction at P has magnitude 25 N.

(4)

(b) Find the magnitude of the force on the rod at A.

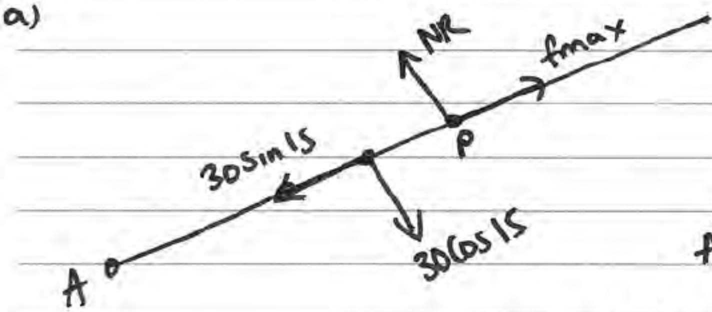
(4)

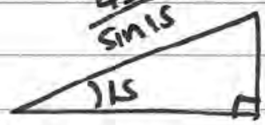
The coefficient of friction between the rod and the peg is μ.

(c) Find the range of possible values of μ.

(4)

a)





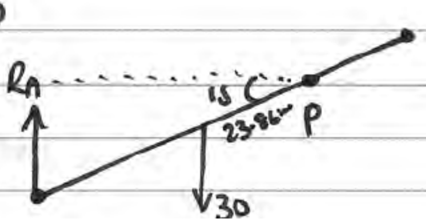
$A\curvearrowright \quad 30\cos 15 \times 150 = NR \times \frac{45}{\sin 15}$

$\Rightarrow \frac{4500}{2} \times 2\sin 15\cos 15 = 45\, NR$

$\Rightarrow \frac{4500}{2} \times \sin 30 = 45\, NR$

$\therefore NR = \frac{4500}{2\times 2\times 45} = 25\,N$ #

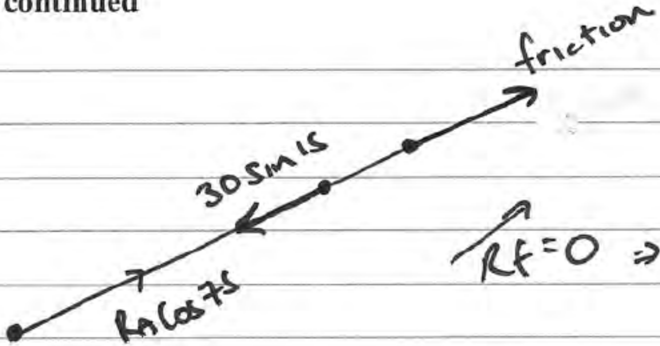
b)



$P\curvearrowright \quad 23.8666\cos 15 \times 30 = R_A \times 173.8666\cos 15$

$RA = 4.12N$

c)



$RF = 0 \Rightarrow \text{friction} = 6.698$

If in limiting equilibrium $f_{max} = \mu NR = 6.698$

$\therefore \mu = 0.267.$

$\therefore 0.267 \leq \mu \leq 1$

7.

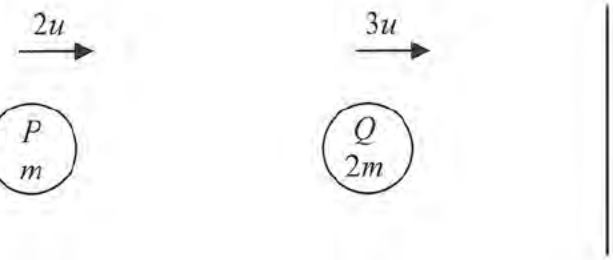


Figure 4

Two smooth particles P and Q have masses m and $2m$ respectively. The particles are moving in the same direction in the same straight line, on a smooth horizontal plane, with Q in front of P. The particles are moving towards a fixed smooth vertical wall which is perpendicular to the direction of motion of the particles, as shown in Figure 4. The speed of P is $2u$ and the speed of Q is $3u$. The coefficient of restitution between Q and the wall is $\frac{1}{3}$. Particle Q strikes the wall, rebounds and then collides directly with P. The direction of motion of each particle is reversed by this collision. Immediately after this collision the speed of P is v and the speed of Q is w.

(a) Show that $v = 2w$.

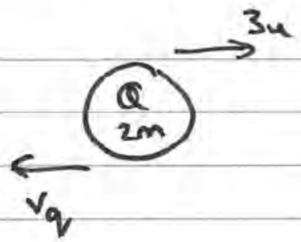
(5)

The total kinetic energy of P and Q immediately after they collide is half the total kinetic energy of P and Q immediately before they collide.

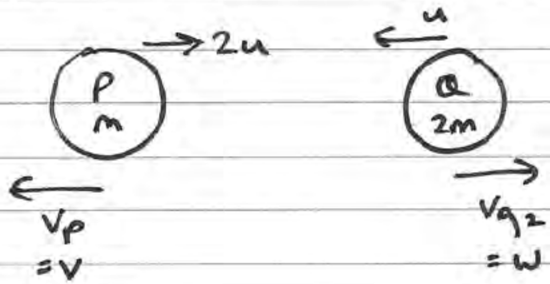
(b) Find the coefficient of restitution between P and Q.

(8)

a)



$e = \frac{V_q}{3u} = \frac{1}{3}$ $\therefore V_q = u$



CLM

$2mu - 2mu = -mV + 2mW$

$\Rightarrow 0 = 2mW - mV$

$\cancel{m}V = 2\cancel{m}w$

$V = 2w$ #

$$KE \text{ before} = \frac{1}{2}m(2u)^2 + \frac{1}{2}(2m)u^2$$

$$\therefore KE \text{ after} = \frac{3}{2}mu^2.$$

$$KE \text{ after} = \frac{1}{2}mv^2 + \frac{1}{2}(m)\left(\frac{1}{2}v\right)^2 = \frac{3}{4}mv^2$$

$$\therefore \frac{\cancel{3}}{2}\cancel{m}u^2 = \frac{\cancel{3}}{4}\cancel{m}v^2 \quad \Rightarrow v^2 = 2u^2$$

$$= \left(\frac{v}{u}\right)^2 = 2$$

$$e = \frac{v+w}{3u} = \frac{\frac{3}{2}v}{3u} \qquad \therefore \frac{v}{u} = \sqrt{2}$$

$$= \frac{1}{2}\left(\frac{v}{u}\right) = \frac{1}{2}\sqrt{2}$$